Eton College King's Scholarship Examination 2012

MATHEMATICS A

Answer Question 1 and as many of the other five questions as you can.

Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all of your working. The use of calculators is permitted.
1. This question is compulsory.

(a) If \( x = -5 \) and \( y = 12 \), evaluate the following, leaving your answers as exact fractions:

(i) \( \frac{8x^2 - y^2}{y - 12x} \)  

(ii) \( \frac{y}{2x} + \frac{x}{y} \)

[3]  [2]

(b) Solve the following inequalities:

(i) \( \frac{2}{3}(x - 3) < 18 \)  

(ii) \( 8 - 3x < 2x - 2 \)

[2]  [2]

(c) Simon and Terry are both told to draw an isosceles triangle which has two angles differing by 15°. They both draw a triangle but find they have drawn ones with different angles from each other. Can they both be correct?  

[3]

(d) Calculate

(i) 15% of £40.00  

(ii) 89% of £111.00

[1]  [1]

(e) A triangle of base length 29.7 cm has area 8.9 cm\(^2\). Find the height of the triangle, giving your answer correct to 2 significant figures.  

[3]

(f) Solve the following equations, leaving your answers as mixed numbers where appropriate:

(i) \( \frac{4 - 3x}{5} = 9 \)  

(ii) \( \frac{2 - 5x}{3} = \frac{5 - 3x}{2} \)

[2]  [3]

(g) In the diagram below, AB and CD are parallel. Calculate the value of \( x \).

[2]

\[ \begin{align*}
\text{A} & \quad \text{B} \\
\text{C} & \quad \text{D}
\end{align*} \]

(h) Solve the following simultaneous equations

\[ \begin{align*}
4x - 3y &= 15 \\
5x + 7y &= 8
\end{align*} \]

[4]
(i) Simplify the following as far as possible:

(i) \((3ab^4)^3\) [2]

(ii) \(\frac{8d^2 - 3d^2}{20d^2}\) [2]

(j) A rectangular field has length 30 m. Its width is half its length.

(i) Find, correct to 3 significant figures, the distance between the opposite corners of the field. [3]

(ii) A second field is 9% less wide but 9% longer. Find the distance, correct to 3 significant figures, between the opposite corners of the second field. [3]

(k) (i) By what do you multiply 8 to get \(2\frac{1}{2}\)? Give your answer as an exact fraction. [2]

(ii) By what do you multiply \(ab\) to get \(\frac{2b^2}{3}\)? [2]

(l) A large company insists that each shareholder invests at least £12,000 in the company. At present, the company has two thousand shareholders and their average investment is £13,040.

(i) How much is their total investment? [2]

(ii) Suppose that 100 new people become shareholders in the company. What is the lowest level to which the average investment could drop? Give your answer to the nearest pound. [2]

(iii) In fact a further \(n\) new people become shareholders and on average they invest £12,320. If the average investment across all the shareholders is now £12,960, find \(n\). [4]

2. The diagram shows four identical circles inside one large circle. The radius of each of the smaller circles is \(\sqrt{2}\) cm.

![Diagram of four circles inside a larger circle]

(a) Prove that the radius of the large circle is \(2 + \sqrt{2}\) cm. [3]

(b) Show that the shaded area is \(2\pi(2\sqrt{2} - 1)\) cm². [4]

(c) Find an exact expression for the total perimeter of the shaded area. [3]
3. This question is about factors of numbers.

(a) Write down all of the 6 factors of 45. [2]

(b) Two whole numbers multiply to give 45. Explain why their sum must be even. [1]

(c) Two whole numbers multiply to give 32. Explain why if their sum is odd, the two numbers must add to 33. [2]

(d) Two whole numbers multiply to give 81. What are the possible values for their sum? [2]

(e) Two whole numbers multiply to give 1,417,176 and add to make 354,298. Find the two numbers. [3]

4. In this question, the diagrams are not drawn to scale.

(a) In the diagram below, the points A, B and C all lie on a straight line and the angle CBD is a right angle. AD and CD are 3 cm and 8.2 cm respectively and AB is 2.4 cm.

![Diagram](image)

(i) Calculate the length BD. [4]

(ii) Calculate the length BC. [4]

(b) Show that \((c - x)^2 = c^2 - 2cx + x^2\). [2]

(c) In the diagram below, P, Q and R lie on a straight line and angle SQR is a right angle. The lengths PS, SR and RP are a, b and c cm respectively and QR is x cm. Use algebra to show that

\[
x = \frac{b^2 + c^2 - a^2}{2c}
\]

![Diagram](image)
5. (a) A postman has ten letters to deliver. Each letter is addressed to exactly one of nine addresses. Explain why at least two letters have the same address on them. \[1\]

(b) 230 Etonians have their birthday during the week starting Monday 30th July. Explain why at least 33 of them must have their birthday on the same day. \[3\]

(c) At a birthday party, thirty five sweets are shared between eight children. Given that each child receives at least one sweet, is it possible for them all to receive a different number of sweets? \[3\]

(d) A rectangle has width 6 cm and height 12 cm. If 9 points are chosen from within the rectangle, explain why two of the points must be at most $\sqrt{18}$ cm away from each other. [Hint: divide the rectangle into squares of equal area.] \[3\]

6. (a) Show by multiplying out that if $(x-A)(y-A) \leq 0$, then $xy \leq A(x + y - A)$ \[2\]

For any set of numbers, the R algorithm is as follows:

- Work out the mean of the numbers: call the answer $A$.
- Replace $x$, the smallest number in the set, and $y$, the largest number in the set, by $A$ and $x + y - A$ respectively. (If there is more than one smallest number, replace the first. If there is more than one greatest number, replace the first.)

For example, for the number set\{1, 4, 6, 9\}, $A = 5$ and so 1 is replaced by 5 and 9 is replaced by 5. Thus the R algorithm replaces \{1, 4, 6, 9\} with \{5, 4, 6, 5\}.

Furthermore, the R algorithm replaces \{5, 4, 6, 5\} by \{5, 5, 5, 5\}.

(b) Show that if the R algorithm is applied twice to \{10, 13, 23, 29, 35\}, the resulting set of numbers is \{22, 22, 23, 20, 23\}. \[2\]

(c) Show further that if the R algorithm is applied 4 times to \{10, 13, 23, 29, 35\}, the resulting set of numbers is \{22, 22, 22, 22\}. \[2\]

(d) Use part (a) to explain why when you apply the R algorithm to a set of numbers, then the new set of numbers will not multiply to give a smaller answer than the original set of numbers. \[2\]

(e) Explain why if you repeat the R algorithm, $A$ remains unchanged. \[1\]

(f) Use your answers to the earlier parts to explain why $10 \times 13 \times 23 \times 29 \times 35 \leq 22^5$. \[1\]

[END OF PAPER]