Eton College King's Scholarship Examination 2013

MATHEMATICS B (One and a half hours)

- Answer as many questions as you can.
- Each of the ten questions carries ten marks.
- Show all of your working.
- Calculators are permitted and some questions will require them.
- Answers should be given to three significant figures unless they can be given exactly or the question states otherwise.
1. In this question, you should use the π button on your calculator.
   a) A circle C has a radius of 25 cm.
      i) Find the area of the circle correct to 5 significant figures.
      A square S has the same area as the circle C.
      ii) Find the perimeter of the square.
   b) A square T has a side length of 10 cm, and a circle K has an area which is twice that of the square T. Find the circumference of the circle K.

2. a) Boys from two classes take a maths test. Class A has 18 boys in it, and Class B has 25 boys in it. The mean score of Class A is 67.5%; the mean score of Class B is 87.2%. Find the mean score of all the boys correct to 3 significant figures.
   b) I cycle 30 km at 18.4 km/hr then jog 30 km at 5.8 km/hr. Find my average speed for the whole journey, correct to 4 significant figures.

3. Let S represent the set of all whole numbers between 1 and 5000 inclusive.
   a) How many numbers in S are divisible by 3?
   b) How many numbers in S are divisible by 7?
   c) Show that 2142 numbers in S are divisible by 3 or 7 or both.
   d) How many numbers in S are divisible by 3, 7 or both but not by 13?

4. In Lo-Maqom, salaries are paid in Lo-Maqomian pounds. Tax is collected annually in the following manner:
   - on the first £5,000 earned, no tax is paid;
   - on the next £5,000 earned, 25% tax is paid;
   - on the next £10,000 earned, 40% tax is paid;
   - on any further income, 60% tax is paid.
   a) Show that a plumber earning £90,000 pays £47,250 tax.
   b) Calculate how much tax is paid by a lawyer earning £15,000.
   c) A construction worker earns £67,500. What percentage of her salary does she pay in tax?
   d) A sailor pays 33% of his salary in tax. What is his salary?
5.  a) List the prime numbers between 50 and 70.

b) Find which of these prime numbers divides 2013 and hence find the full factorization of 2013.

c) Multiply out and simplify \((x - y)(x + y)\).

d) Two whole numbers, which differ by no more than 10, have squares which differ by 2013. Using your results from parts (c) and (b), find the smaller of these numbers.

[Working must be shown; no credit will be given for answers found by trial and error]

6. In Eton General Stores, they have just started to stock cats, mats and hats; all cats are the same price as each other, all hats are the same price as each other, and all mats are the same price as each other.

   In the Store, Ashley pays £125 for a cat and a hat, whilst Bernard pays £94 for a hat and a mat.

   a) How much more is a cat than a mat?

   Charles claims he bought 10 cats and 9 mats for £690.

   b) Assuming this is true, how much is a cat?

   Charles is a notorious liar, and it wasn’t true. Derek, who is truthful but forgetful, says he can’t remember exactly how much he paid for 4 mats and a hat on Monday, or how much he paid on Tuesday for 5 cats, but he remembers he paid £145 more on Tuesday than on Monday.

   c) Find the actual price of a cat.

7. In this question, the letters \(n\) and \(m\) represent whole numbers.

   a) For each expression, write down whether it is always divisible by 3, never divisible by 3, or sometimes divisible by 3. In each case, give a reason:

   i) \(21n^2\)
   ii) \(33n + 15m\)
   iii) \(5n + 7m\)
   iv) \(9m + 6n + 13\)

   b) Multiply out and simplify:

   i) \((3n+1)(3n+1)\)
   ii) \((3n-1)(3n-1)\)
   iii) \((3n)^2\)

   c) Explain clearly why none of the square numbers (1, 4, 9, 16, …) is one less than a multiple of 3.
8. Four different pumps A, B, C and D can be used individually or together to fill up identically sized oil tankers; each pump works at its own constant rate. Pump A alone will fill a tanker in 30 minutes and pump B alone fill a tanker in 20 minutes.

a) If both pump A and pump B are used together, show that it will take 12 minutes to fill a tanker.

Pump C alone fills a tanker in 3 hours, and pumps A, C and D together fill a tank in 21 minutes and 36 seconds.

b) How long would it take pump D alone to fill a tanker, in hours and minutes?

9. a) In the diagram on the right, the triangle $ABC$ has an area of $314 \text{ cm}^2$. The line $CD$ has length $h \text{ cm}$ and is perpendicular to the line $AB$, which has length 112 cm.

Find $h$ to 3 significant figures.

b) In the diagram below right, triangles $ABD$ and $AED$ are both isosceles; the angles $\angle ABD$, $\angle ADB$, $\angle ADE$ and $\angle AED$ are all equal, the length $AB$ is 7225 mm and the length $DE$ is 6120 mm. The point $C$ lies on $BD$ equidistant from $B$ and $D$; the line $BE$ intersects $AD$ at the point $F$.

i) Find $AC$.

ii) Find the area of triangle $ABD$.

iii) Write down the size of angle $\angle AFB$.

iv) Find the length $BE$. 

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[Turn over]
10. Consider the following expression for $S$, involving consecutive multiples of 7:

$$S = 7 + 14 + 21 + 28 + \ldots + 1386 + 1393 + 1400$$

a) How many numbers are being added together?

Because all the numbers are being added, they can be re-ordered in the following way:

$$S = (7 + 1400) + (14 + 1393) + (21 + 1386) + \ldots + (693 + 714) + (700 + 707)$$

b) Calculate each of the five brackets shown, and write down what pattern you notice.

c) Explain clearly why this pattern should hold for the remaining brackets which are not shown.

d) Hence show that $S = 140700$

e) Use a similar method to find

i) $T = 3 + 6 + 9 + 12 + \ldots + 2994 + 2997 + 3000$

ii) $U = 9 + 13 + 17 + 21 + \ldots + 2009 + 2013$

iii) $V = 3 + 8 + 13 + 18 + \ldots + 2008 + 2013$

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