

Eton College King's Scholarship Examination 2016

MATHEMATICS B

(One and a half hours)

Candidate Number: _____

*Write your candidate number, **not your name**, in the space provided above and on **each** sheet of the question paper.*

Answer as many questions as you can.

Each of the eight questions is worth the same number of marks.

Show all your working.

Calculators are allowed.

ANSWERS SHOULD BE WRITTEN ON THE QUESTION PAPER

ADDITIONAL MATERIALS: CALCULATOR (*provided by candidate*)

Do not turn over until told to do so.

1. a) Solve the equation:

$$0.78(65 - x) + 0.22x = 2.4$$

- b) In a certain country, there are two political parties, Crimson and Scarlet. However, a quarter of Scarlet voters are embarrassed they vote Scarlet and pretend they vote Crimson in surveys, and similarly a fifth of Crimson voters pretend they vote Scarlet in surveys. In a very large survey, 51.4% of those asked *said* they would vote Crimson, and the rest said they would vote Scarlet. What percentage of voters would one expect *actually* to vote Crimson?

2. A marathon was due to start at midday, but Mr Tortoise was allowed to start at 7am as he can only walk, which he does at a constant speed of 3mph. At midday, Mr Hare sets off running at a constant speed of $10\frac{1}{2}$ mph.

a) At what time does he overtake Mr Tortoise?

b) At what time is he exactly two miles ahead of Mr Tortoise?

3. a) Expand the brackets and simplify:

i) $(x^2 + y^2)^2$

ii) $(x^2 - y^2)^2$

b) A right-angled triangle has a hypotenuse of length $x^2 + y^2$ and another side of length $x^2 - y^2$ where x and y are positive numbers with x greater than y . Use algebra and your results from above to find the length of the third side. Give your answer in simplified form in terms of x and y .

- c) I am trying to draw a right-angled triangle with a hypotenuse of 221cm and the other two sides each being a whole number of centimetres long.
- i) By writing 221 as the sum of two square numbers, find one possible solution for the length of the other two sides.
- ii) By factorizing 221, find another possible solution for the length of the other two sides.

4. There is a certain whole number between 100 and 999 whose digits are all different and with the following property: the number equals eleven times the sum of all the digits.
- a) Explain why the digits must add to *more* than 9.

 - b) Explain why the digits must add to *no more* than 24.

 - c) Use your answer to part (b) to show that the digits must in fact add to no more than 19.

 - d) Find the number and the sum of its digits.

5. a) Show that $x(x+y) + y(x+y) = (x+y)^2$.

b) x , y and z are positive numbers such that the following three equations hold:

$$x^2 + xy + xz = 56$$

$$xy + y^2 + yz = 98$$

$$xz + yz + z^2 = 42$$

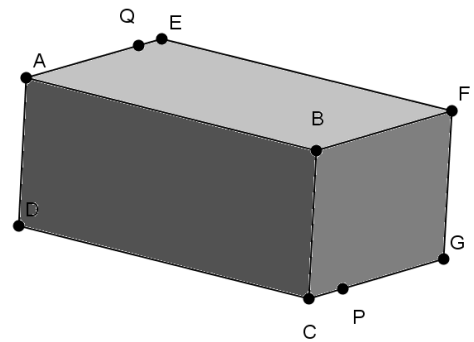
i) Factorize completely the left-hand side of each equation.

ii) Show that $(x+y+z)^2 = 196$.

iii) Hence find each value x , y and z .

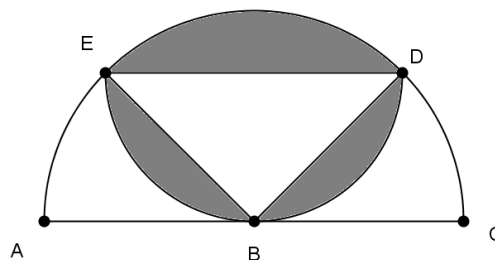
6. a) A rectangle has one side of length 24cm and a diagonal of length 25cm. Find the area of the rectangle.

- b) A solid stone cuboid $ABCDEFGH$ is shown
[Note: point H is hidden]. Face $BFGC$ and face $AEDH$ are squares of side length 24cm; side AB is 75cm. A certain point P lies on side CG , 2cm from C and a point Q lies on side AE , 2cm from E . A chalk line is drawn on the surface of the cuboid from point P to point Q . Find the shortest possible length of such a line.



[Space for working for question 6]

7. In the diagram, EBD is 90° ; lines ED and ABC are parallel. $AEDC$ is a semicircle with a centre at B ; EBD is a semicircle. BD has length $\sqrt{2}\text{cm}$.
- a) Find the length ED .



- b) Find the area of triangle BED .

- c) Find the shaded area, giving your answer to 3 significant figures.

8. Circle the most suitable next term in the sequence from the list of options and *briefly* explain your answer.

a) 81, 108, 144, 192, 256

OPTIONS:

A: $339\frac{5}{6}$ B: 342 C: $340\frac{1}{4}$ D: $341\frac{1}{3}$ E: 340

REASON:

b) 1, 1, 3, 5, 11, 21, 43, 85

OPTIONS:

A: 155 B: 171 C: 147 D: 169 E: 197

REASON:

c) $1, 1, 1\frac{1}{3}, 2, 3\frac{1}{5}, 5\frac{1}{3}, 9\frac{1}{7}, 16, 28\frac{4}{9}$

OPTIONS:

A: $63\frac{1}{10}$ B: 32 C: $53\frac{1}{6}$ D: $33\frac{1}{10}$ E: $51\frac{1}{5}$

REASON:

d) $\frac{1}{2}, \frac{5}{6}, 1\frac{1}{30}, 1\frac{37}{210}, 1\frac{617}{2310}$

OPTIONS:

A: $1\frac{10331}{30030}$ B: $1\frac{7907}{30030}$ C: $1\frac{17107}{30030}$ D: $1\frac{12641}{30030}$ E: $1\frac{18839}{30030}$

REASON:

END OF PAPER

