Eton College King's Scholarship Examination 2024

MATHEMATICS B

(One and a half hours)

Candidate number:**.....**

<u>Please write your candidate number on EVERY sheet.</u>

Please answer on the paper in the spaces provided.

There are 8 questions: each one is worth 10 marks.

Calculators are allowed, but you should show all your working.

Do not turn over until told to do so.

1.

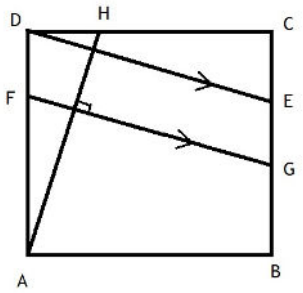
(a) 13 apricots, 7 bananas and 20 cherries cost £3.29.

10 apricots, 11 bananas and 21 cherries cost £3.71.
Show that with this limited amount of information, it is possible to find the difference in price between one apricot and one banana, and find this difference.

(b) In my drawer I have knives, forks, dessert spoons and teaspoons.
Each knife weighs 75g, each fork 65g, each dessert spoon 90g and each teaspoon 25g.
In total, I have 69 items of cutlery, weighing 4.3kg in total.
If I have twice as many knives as dessert spoons, and three fewer forks than knives, how many teaspoons do I have?

2.

(a) *ABCD* is a square.
FG is parallel to *DE*.
AH is perpendicular to *FG*.



Explain **very briefly** in terms of transformations why

(i) *FG* and *DE* are equal in length.

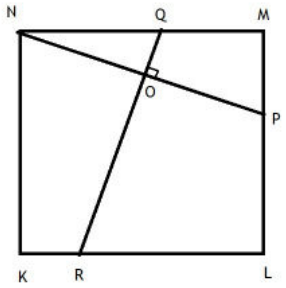
(ii) *AH* and *FG* are equal in length.

(b) *KLMN* is a square of side length 72cm.

Lines *NP* and *RQ* are perpendicular and intersect at *O*, which is the midpoint of *NP*.

MP is of length 30cm.

Find the length of *OR* as a mixed fraction.



3.

(a) The front wheels of farmer Barley's tractor are $2\ ^{1}/_{3}$ metres in circumference; the back wheels are $4\ ^{2}/_{11}$ metres in circumference. Barley drives the tractor in a straight line without slipping.

(i) The tractor travels 100m. Find, as a mixed fraction, the number of revolutions each wheel has made.

(ii) How far will the tractor have travelled when the front wheels have made 45 more revolutions than the back wheels? Give your answer as a mixed fraction.

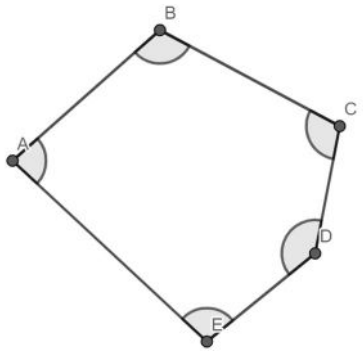
(b) Olaf is making Firecake. His first attempt, Mixture F, contains one eighth brimstone and one eighth charcoal and the remainder saltpetre. His second attempt, Mixture U, contains 15% brimstone, 22.5% charcoal and the remainder saltpetre. He knows that the perfect Firecake contains brimstone and charcoal in the ratio 3:4. How many barrels of Mixture U should he mix with six barrels of Mixture F in order to produce the perfect mixture, and what is the ratio of brimstone : charcoal : saltpetre in the resulting Firecake?

Give your answer in the form $B : C : S$, where B, C and S are integers.

4. This question concerns shapes constructed from straight line segments. Formal geometrical proof is not expected in this question; your reasoning may be very brief.

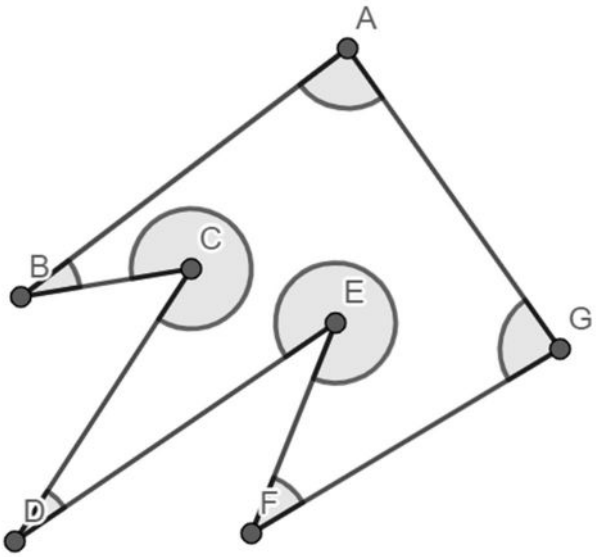
In each case, find an equation which relates the **marked** angles at the labelled vertices to a multiple of 180°.

Example:

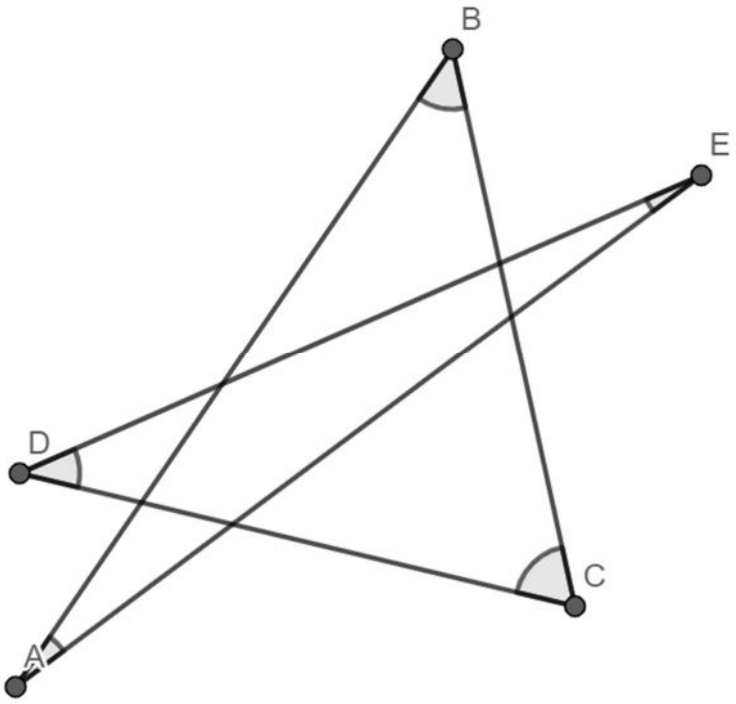


$A + B + C + D + E = 540°$

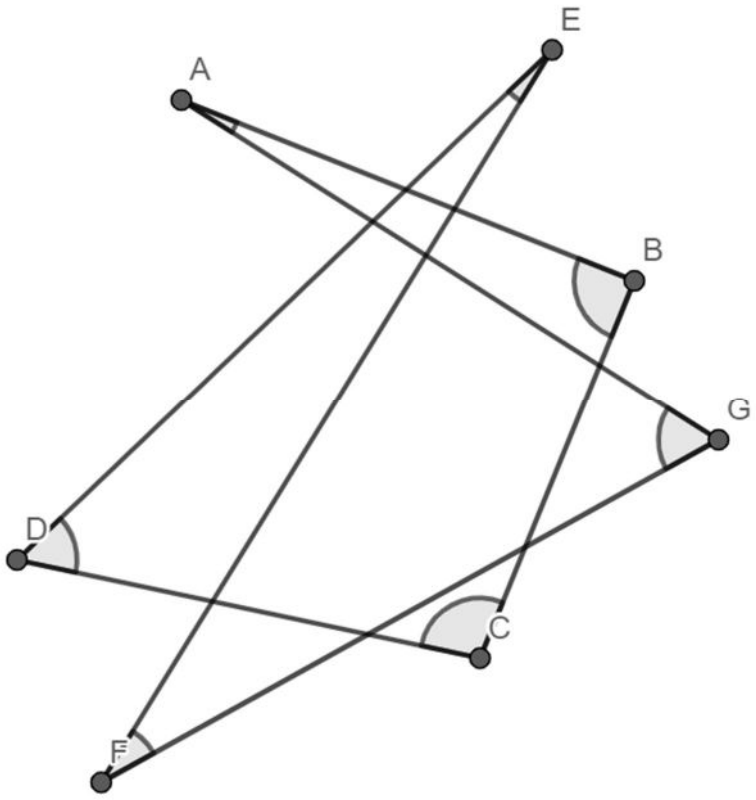
(a)



(b)



(c)



5. Lord Bessborough has an ordinary watch with three hands. It is unusual only in that:

- when the second hand points to twelve, the watch glows for an instant;
- when the minute hand is directly on top of the hour hand, the watch beeps.

(a) His steam train, Tessie, always covers the eight miles of track round his estate at an average speed of 33 miles per hour. Today, Tessie started out just as Lord Bessborough's watch glowed and finished just as it beeped. At what time did Tessie set out?

In Lord Bessborough's absence, his butler Brickett introduces a metrification of time. Days are no longer divided into 24 hours but into 10 new-hours. Each new-hour is divided into 100 new-minutes. Each new-minute is divided into 100 new-seconds. A new time written as 3:71:06 means 3 new-hours, 71 new-minutes and 6 new-seconds after midnight.

(b) Brickett has a dental appointment at 6:05:00 new time. Find this time to the nearest minute in traditional time, using the 24-hour clock.

(c) When Lord Bessborough returns, Brickett converts his watch to the new time system, the numbers on the watch face being arranged as shown.

Lunch is served when the watch first beeps after midday.

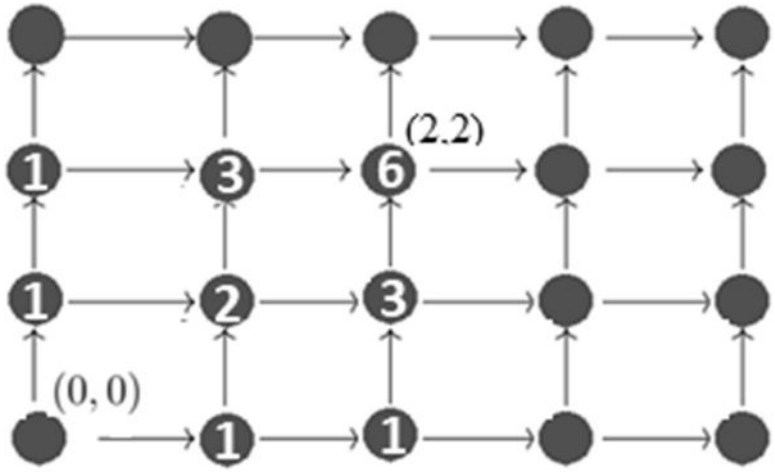
Find this time in both new-time and traditional time, to the nearest new-second and the nearest traditional second respectively.

6. The trees of the Wild Wood are planted on a square grid system, with the rows and columns of trees aligned North to South and East to West, the distance between any two successive trees being a yard.

Owl thinks of his house as occupying grid position (0,0). Relative to this origin, his friend Bat lives at (2,2).

Owl likes to move only one tree at a time, and even then only between nearest neighbours.

He counts that there are six distinct routes he can take to travel the four yards to see his friend, assuming he always travels North or East only.



(a) Assuming that he only ever flies North or East, how many distinct routes could he take to visit

(i) Cat, who lives at grid reference (4, 3)?

(ii) Rat, who lives at grid reference (1, R), where R is a positive integer?

(iii) Cat, given that he must go via Bat's house?

(b) Owl sets out from home and only ever flies North or East.

(i) How many different destinations could he reach after travelling

(A) exactly 5 yards?

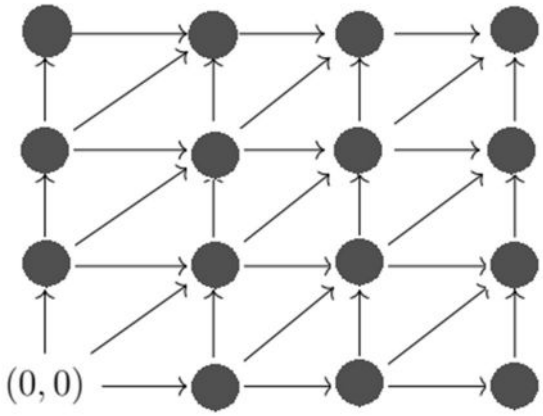
(B) exactly M yards, where M is a positive integer?

(ii) How many distinct routes could he have used to travel

(A) exactly 5 yards?

(B) exactly M yards?

(c) If Owl is feeling bold, he will also fly from one tree to a diagonal nearest neighbour, as shown.



Assuming that he only ever flies North, East or North-East, how many distinct routes are there now for him to visit

(i) Cat?

(ii) Rat?

(iii) Cat, given that he must go via Bat's house?

7. The *ABab* programming language contains only four commands: **A, B, a** and **b.** Programs are written by joining together any number of these commands in any order, eg

ABaaBabA is a program of length 8
BBBBa is a program of length 5
A is a program of length 1

However, if the commands **A** and **a** are immediately next to each other, they exactly cancel each other out. Likewise, **B** and **b** cancel exactly when adjacent. This is seen as an error in the code. Novice programmers might write programs like **AAaBa**, **bBAaB** or **aabBA**, but these are corrected to **ABa**, **B** and **a** respectively.

(a) Show that **BaABbbb** is corrected to just **b**, showing the cancellation carefully.

Programs such as **bBAa** and **BaABbb**, which are corrected to the program of length 0, are referred to as vanishing programs.

(b) List all the non-vanishing programs of length 2 in *ABab*.

A new language, *DEFdef*, contains six commands. Programs are written in a similar way to those in *ABab*, with adjacent pairs **dD**, **Dd**, **eE**, **Ee**, **fF** and **Ff** cancelling.

(c) How many non-vanishing programs of length 2 are there in *DEFdef*?

Programs in *DEFdef* can be rewritten in *ABab* by making the following replacements:

Replace **D** with **AA**
Replace **E** with **BB**
Replace **F** with **AB**
Replace **d** with **aa**
Replace **e** with **bb**
Replace **f** with **?**

(d) Using the requirement that a vanishing program in *DEFdef* must vanish when rewritten in *ABab*, find the two-command replacement for **f**.

(e) Using this system, which *DEFdef* programs, when rewritten in *ABab*, become

(i) **Ab**

(ii) **aB**

(iii) **ab**

(iv) **BA**

8. Firemen Ashe, Burns and Chard have taken two sets of examinations, in the hope of being promoted up the ranks of the firehouse staff. In each individual examination, the firemen are ranked first, second and third, with each rank receiving a distinct numerical score.

(a) In the first set of examinations, first place scores 6 marks, second place scores 3 marks and third place scores 1 mark in each exam. Each fireman's score is totalled. For each of the following lists of total scores, either show a way in which it might have been achieved or explain **briefly** why it is impossible.

(i) A: 97 B: 24 C: 34

(ii) A: 79 B: 12 C: 49

(iii) A: 91 B: 30 C: 29

(iv) A: 58 B: 21 C: 21

(b) In the second set of examinations, first place scores f marks, second place scores s marks and third place scores t marks in each exam, where f, s and t are distinct positive integers with $f > s > t$.

The total of the marks obtained by each candidate is: Ashe: 20, Burns: 10, Chard: 9.
If Burns came first in Hose Control, who came second in Klaxon Ringing?
Explain your reasoning clearly.

[END OF PAPER]